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TESTING RATIONALITY IN CONTINGENT VALUATION STUDIES

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Abstract

In a world characterized by one unrationed good (ie, market good), and one rationed, such as most of environmental goods, we obtain sufficient conditions for the existence of an underlying rational preference structure based on a system of mixed demand functions. The implications for CV studies are derived. For policy purposes, in particular in what concerns welfare evaluations, our results can be of great relevance.

Keywords: rationality; mixed demand functions; preference-based versus choice-based approach

I Introduction

One of the goals of applied demand analysis is to make welfare evaluations for policy purposes. When making these welfare evaluations, based on the data available, the researcher relies on the neoclassical theory of preferences. In this context, the question to be addressed is how to legitimate the use of the data in order to have welfare significance. To this end, the first step consists of testing for the existence of an underlying rational preference structure.

In particular, Contingent Valuation Methods (CVM), namely in the discrete-choice format, have gained popularity for eliciting economic value of nonmarket resources and public goods. When rigorously conceived and cautiously interpreted, these surveys can provide useful information about the characteristics of demand for nonmarket goods. An important argument that has been used in the literature against contingent valuation is that it is not based in revealed preferred information, and therefore, it is not supported by the neoclassical theory of preferences.[1] As a result, validation of empirical applications of CVM has received considerable attention in the literature.

In a world characterized by market goods and environmental goods for which markets, in general, do not exist, the researcher is confronted with the data available, market observed/revealed preferred (RP) and/or generated by hypothetical surveys/stated data (SP). Therefore, we argue that the individual's choice behavior should be treated as the primitive feature, and the approach to modelling individual choice behavior is a *choice-based approach*, in contrast to the *preference-based* one. In this last case, the decision maker's tastes, as summarized in his *preference relation*, are taken as the primitive characteristic of the individual, while in the choice-based approach the primitive feature is the individual

[1] See Hanemann (1994).

choice behavior.[2] In this context, the first question to be addressed in any empirical analysis should be how to legitimate the use of the data to secure welfare significance, which requires testing for the existence of an underlying rational preference structure. This is the purpose of our study.

When working with real market data (RP), we observe a rule that assigns a set of chosen consumption bundles to each price-income pair (p,w), that is, a direct individual walrasian or ordinary demand function. Typically, this is what occurs with indirect methods for valuing environmental amenities, as travel cost methods. In this context, a differentiable walrasian demand function $x(p,w)$, satisfying Walras' law and homogeneity of degree zero in prices ($p \gg 0$) and income (w), is generated from the maximization of a quasi-concave utility function if and only if the matrix of the substitution effects $S(p,w)$ (ie, the Slutsky matrix) is symmetric (S) and negative semi-definite (NSD), for all (p,w). In Cunha-e-Sá and Ducla-Soares (1996), the conditions for the existence of an underlying rational preference were examined in a particular empirical relevant case.

When the data available to the researcher is stated data (SP), and in contrast to the revealed data case (RP), we do not observe a walrasian demand function. The surveys have gained popularity for eliciting the economic value of nonmarket resources and public goods. Depending on the question asked, the Willingness To Pay (WTP) or Willingness To Accept (WTA) for marginal changes in the availability of the environmental good can be recovered. In these cases, individual Hicksian or compensated inverse demand functions can be derived.

The consumers make their choices over goods that are either freely available in the market or for which they face constraints, as when they are quantitatively rationed. In particular, environmental and other public goods fall into this class. Therefore, all the adjustments in the optimal choices will reflect the substitutability and/or complementarity between these two different kinds of goods. The behavior of consumers in this more general setting is characterized by demand relationships that are functions of a mixed set of prices and quantities. Assuming that the rationed goods are always consumed optimally, that is, that the price imputed to the rationed good is the one for which the ration level will equal the optimal quantity in the unrationed problem (ie, the virtual price of the rationed good), the appropriate system of

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[2] See Mas-Colell et al. (1995).

demand functions to describe the choice behavior of individual consumers is a system of mixed demand functions. These more flexible demand functions were introduced by Samuelson (1965), and are also implied by consumer utility maximization.

Given that the structure of the data available in this more general context, that is, with market and nonmarket goods, leads to a system of both direct and inverse compensated demands, the appropriate setting is a system of mixed demand functions. In order to test for rationality in this context, we have to carefully examine the properties of the matrix of the substitution effects. In this paper we derive sufficient conditions under which an underlying rational preference exits in a particular empirical relevant case, that is, a world of two goods, one unrationed and one rationed, or the environmental good. The implications for the design of future Contingent Valuation studies are examined.

The rest of the paper is organized as follows. Section 2 states the consumer's choice problem under demand rationing, from a dual perspective. The system of mixed demand functions is derived. In Section 3, the relationship between the matrix of the substitution effects (ie, the Slutsky matrix) for the nonconstrained and the constrained problems is characterized. Proposition 1 summarizes these results. Moreover, the implications of testing rationality for a world of one unrationed good and one rationed are examined for CV studies. Finally, in Section 4 the main conclusions of the paper are presented.

2 The Cost Minimization Model under Demand Rationing

In a world of two goods i = 1,2, where i=1 is unrationed, and i=2 is rationed, the rationed cost minimization problem, based on the concept of *virtual price* of the rationed good (p_2^v), that is, the price for which the rationed quantity would be optimal, can be stated as

$$\underset{\{q_1,\bar{q}_2\}}{Min}\; p_1 q_1 + p_2^v \bar{q}_2$$

s.t.

$$u(q_1,\bar{q}_2) \geq \bar{u}$$
$$q_1 \geq 0$$
$$\bar{q}_2 \geq 0$$

where p_1 and q_1 represent the price and quantity of good 1, respectively, and $\bar{q}_2$ is the fixed (nonnegative) quantity of the rationed good. Moreover, $u(.)$ is a well-behaved direct utility function. The solution to this problem is given by the following system of compensated demand functions:

$$q_1 = q_1(p_1, p_2^v, \bar{u})$$
$$\bar{q}_2 = \bar{q}_2(p_1, p_2^v, \bar{u})$$

The corresponding cost function is

$$C(p_1, p_2^v, \bar{u}).$$

The matrix of substitution effects, that is, the Slutsky matrix is

$$S_c = \begin{bmatrix} \dfrac{\partial q_1}{\partial p_1} & \dfrac{\partial q_1}{\partial p_2^v} \\ \dfrac{\partial \bar{q}_2}{\partial p_1} & \dfrac{\partial \bar{q}_2}{\partial p_2^v} \end{bmatrix}$$

The existence of an underlying rational preference structure is intimately related to the properties of the matrix S_c, that is, negative semidefiniteness (NSD) and symmetry (S).

In a world characterized by private goods and environmental goods for which, in many cases, a market does not exist, the choice rules that represent the behavior of consumers in this more general setting are characterized by demand relationships that are functions of a mixed set of prices and quantities, that is, a system of mixed compensated demand functions. By applying the Sheppard's lemma and the Implicit Function Theorem, the corresponding system of mixed compensated demand functions can be stated as follows:[3]

$$q_1^m = q_1^m(p_1, \bar{q}_2, \bar{u})$$
$$p_2^v = p_2^v(p_1, \bar{q}_2, \bar{u})$$

where $\bar{q}_2$ is an optimal choice.

In this more general setting, the corresponding matrix of substitution effects is:

[3] These functions are, respectively, homogeneous of degree zero and one on the prices of the unrationed good. Moreover, Walras's law is satisfied.

$$\hat{S}_c = \left[\hat{s}_{ij}\right] = \begin{bmatrix} \frac{\partial q_1^m}{\partial p_1} & \frac{\partial q_1^m}{\partial \bar{q}_2} \\ \frac{\partial p_2^v}{\partial p_1} & \frac{\partial p_2^v}{\partial \bar{q}_2} \end{bmatrix}$$

3 Testing for Rationality in a Two Goods Mixed World

3.1 Relationship between S_c and $\hat{S}_c$

In this section, the relationship between S_c and $\hat{S}_c$ is established. Lemma 1 in Madden (1991) establishes this relationship for the general case, expressing $\hat{S}_c$ in terms of the elements of S_c. Differently from Madden (1991), by expressing the elements of the S_c matrix in terms of the elements of $\hat{S}_c$, we obtain:

$$S_c = \left[s_{ij}\right] = \begin{bmatrix} \frac{\partial q_1^m}{\partial p_1} - \frac{\frac{\partial q_1^m}{\partial \bar{q}_2}\frac{\partial p_2^v}{\partial p_1}}{\frac{\partial p_2^v}{\partial \bar{q}_2}} & \frac{\frac{\partial q_1^m}{\partial \bar{q}_2}}{\frac{\partial p_2^v}{\partial \bar{q}_2}} \\ -\frac{\frac{\partial p_2^v}{\partial p_1}}{\frac{\partial p_2^v}{\partial \bar{q}_2}} & \frac{1}{\frac{\partial p_2^v}{\partial \bar{q}_2}} \end{bmatrix}$$

As far as the bottom right-hand element of S_c is invertible, $\hat{S}_c$ can be directly obtained from S_c. In order to guarantee the existence of an explicit solution for the mixed problem, the Implicit Function Theorem requires that element to be different from zero.

Our goal is to derive the conditions (to be tested) under which an underlying rational preference structure exists. These conditions are imposed on the elements of $\hat{S}_c$, rather than on those of S_c. From an empirical point of view, two main reasons justify this procedure. First, the estimates of the elements of $\hat{S}_c$ are directly obtained in the estimation process, and second, these elements are simpler algebraic expressions when compared with the corresponding ones in S_c. Moreover, the properties of S_c are wellknown, that is, symmetry (S) and negative semi-definiteness (NSD). Therefore, and, in contrast to Madden (1991), S_c is expressed in terms of the elements $\hat{S}_c$.

In order to obtain the conditions on the elements of $\hat{S}_c$ under which the underlying rational preference structure exists, we will take advantage of (i) the homogeneity of degree one in prices of the direct compensated demand function of the unrationed good, and (ii) the relationship between S_c and $\hat{S}_c$ as well as the properties of S_c.

The results are summarized in Proposition 1, and in Corollary 1.

Proposition 1: If *(i)* $\left(\dfrac{\partial q_1^m}{\partial \bar{q}_2} = -\dfrac{\partial p_2^v}{\partial p_1}\right)$

(ii) $\left(\dfrac{\partial p_2^v}{\partial \bar{q}_2} < 0\right)$

(iii) $\hat{s}_{11} = \dfrac{\partial q_1^m}{\partial p_1} = 0$

then, all the desirable properties of S_c *are guaranteed. Moreover, by homogeneity of degree zero in prices of the direct compensated demand function of the unrationed good,* $\hat{s}_{12}$ *has to be negative, (or* $\hat{s}_{21} < 0$*).*

Proof: By inspection of S_c, it follows that condition (i) is implied by symmetry of S_c. Negative semi-definiteness of S_c implies negativeness of the direct substitution effects, as stated in condition (ii). This fact follows immediately by inspection of the diagonal elements of S_c. Condition (iii) is implied by singularity of S_c, since it can be shown that

$$\hat{s}_{11} = \frac{\det(S_c)}{\hat{s}_{22}}$$

Finally, by homogeneity of degree zero in prices of the direct compensated demand functions of the unrationed good, that is, $\hat{s}_{12} = \dfrac{\partial q_1^m}{\partial \bar{q}_2} < 0$. In fact,

$$\frac{\partial q_1}{\partial p_1}p_1 + \frac{\partial q_1}{\partial p_2^v}p_2^v = 0$$. As $\frac{\partial q_1}{\partial p_1^v} = \frac{\frac{\partial q_1^m}{\partial \bar{q}_2}}{\frac{\partial p_2^v}{\partial \bar{q}_2}} = \frac{\hat{s}_{12}}{\hat{s}_{22}}$, and given (ii), $\hat{s}_{12}$ has to be negative,

or $\hat{s}_{21}>0$. Note that $\hat{s}_{12} = \frac{\partial q_1^m}{\partial \bar{q}_2} < 0$, or $\hat{s}_{21}>0$, reflects a substitutability relationship between the two goods in a classification different from the Slutsky one. This new classification was developed in Madden (1991), in the context of a mixed world. If (i), (ii), and (iii) hold, then the matrix S_c is symmetric, and negative semi-definite, with all the principal minors non singular, except for S_c itself. Q.E.D

The results in Proposition 1 have some implications to the matrix $\hat{S}_c$, as stated in Corollary 1.

Corollary 1: *Given Proposition 1, the matrix $\hat{S}_c$ has the following properties:*

(i) skew-symmetric, that is, $\hat{s}_{12} = -\hat{s}_{21}$, with $\hat{s}_{21}>0$;

(ii) $\hat{s}_{11} = 0$;

Proof: The implications to the elements of $\hat{S}_c$ are the following:

(i) Symmetry of S_c implies skew-symmetry of the nondiagonal elements. Homogeneity of degree zero in prices of the direct compensated demand function of the unrationed good implies that $\hat{s}_{21}>0$.

(ii) The singularity of S_c is equivalent to $\hat{s}_{11} = 0$, as $\hat{s}_{11} = \frac{\det(S_c)}{\hat{s}_{22}}$, but it does not imply singularity of $\hat{S}_c$. Q.E.D

3.2 Implications for CV Studies

In the context of the environmental economics literature, the majority of empirical studies dedicated to value environmental assets estimate a demand function or a marginal willingness to pay (WTP) function for changes in the availability, or in the quality of a given environmental asset. The arguments of the demand function are, besides the availability, or the quality of the environmental asset, the set of individual attributes. In general, these studies involve two goods, one market good and one rationed good, that is, the environmental good.

From an empirical perspective, given Proposition 1, and Corollary 1, four tests have to be performed for rationality:

(i) $\hat{s}_{12} = -\hat{s}_{21}$;

(ii) $\hat{s}_{21} > 0$;

(iii) $\hat{s}_{22} < 0$;

(iv) $\hat{s}_{11} = 0$;

This has important implications for the design of the empirical study. In general, only an estimate for the marginal willingness to pay for a marginal change in the availability or the quality of the environmental good ($\hat{s}_{22}$) is provided. However, even in the cases in which it is not possible to reject the hypothesis that $\hat{s}_{22}$ is negative, more is needed, in particular, tests on (i), (ii) and (iv). These tests are not performed, as the required information is not typically provided by the surveys in contingent valuation studies. In fact, not only it is not possible to disentangle the marginal effect of a change in the price of the market good on the marginal willingness to pay for a marginal change in the availability or the quality of the environmental good ($\hat{s}_{21}$), but also, an estimate of the direct compensated demand function for the unrationed good is not available. Therefore, for rationality purposes these results suggest that a new design of the surveys is required.

4. Conclusions

The purpose of this study is to derive testable conditions for the existence of a rational preference structure, in a world characterized by the presence of both market and nonmarket goods, such as most of the environmental goods. Without rationality, welfare evaluations cannot be undertaken. As they represent one of the main goals of the majority of the empirical applications in this field, failure to satisfy it would definitely compromise the chance of making use of the estimates for policy purposes. The conditions that we have obtained are testable in the sense that they depend directly on the estimated coefficients of a mixed system of demand functions, and, therefore, observable to the researcher.

We apply the methodology developed in the paper to a particular empirical relevant case, that is, the world of two goods, where one is a market good and the other is rationed. We show that for rationality purposes a new design of the questionnaires and surveys to be used in future contingent valuation studies is required.

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